Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Let $y=mx+\sqrt{(a^2m^2+b^2)}$, $y=m'x+\sqrt{(a^2m'^2+b^2)}$ be the equation of the tangents parallel to two conjugate diameters. But between m and m' there is the well known relation: $mm'=-(b^2/a^2)$, whence $m'=-(b^2/a^2m)$. Substituting this in the second of the above equations, we have $a^2ym+b^2x=ab\sqrt{(a^2m^2+b^2)}$, and dividing the last equation by $y-mx=ab\sqrt{(a^2m^2+b^2)}$, we obtain

$$m=\frac{b}{a}\cdot\frac{ay-bx}{ax+bx};$$

and substituting in the latter equation, we finally get $a^2y^2+b^2x^2=2a^2b^2$, the equation of a concentric ellipse with the semi-axes $a\sqrt{2}$ and $b\sqrt{2}$.

391. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Burghfield, England.

An ellipse is inscribed in the triangle of reference and has one focus at (sec$A$, sec$B$, sec$C$). Find the other focus and the sum of the squares of the axes of the ellipse.

Solution by WILLIAM HOOVER, Ph. D., Athens, Ohio.

In trilinear coördinates, let $(\alpha_1, \beta_1, \gamma_1)$, $(\alpha_2, \beta_2, \gamma_2)$ be the two foci. Then the first is found by

$$\alpha_1\cos A=\beta_1\cos B=\gamma_1\cos C\ldots(1),$$
$$\text{and } a\alpha_1+b\beta_1+c\gamma_1=2\triangle\ldots(2),$$

or, $\alpha_1=2R\cos B\cos C$, $\beta_1=2R\cos A\cos C$, and $\gamma_1=2R\cos A\cos B\ldots(3)$,

R being the radius of the circum-circle.

If b_1 be the semi-minor axis of the ellipse,

$$\alpha_1\alpha_2=b_1^2=\beta_1\beta_2=\gamma_1\gamma_2\ldots(4),\text{ or,}$$
$$\cos B\cos C.\alpha_2=\cos A\cos C.\beta_2=\cos A\cos B.\gamma_2\ldots(5),$$

which with (3) gives

$$\alpha_2=R\cos A,\ \beta_2=R\cos B,\ \gamma_2=R\cos C\ldots(6).$$

(4) now gives $b_1^2=2R^2\cos A\cos B\cos C\ldots(7)$.

Also, d=the distance between the ortho-center and circum-center, is given by

$$d^2=R^2(1-8\cos A\cos B\cos C)\ldots(8).$$

If $2a_1$ be the major axis of the ellipse, $a_1^2=\frac{1}{4}d^2+b_1^2=\frac{1}{4}R^2$, or $2a_1=R$, and determining $4a_1^2+4b_1^2$.

392. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

A tangent to a curve at any point P cuts the tangent and the normal at a fixed point O in the points M and N, and the rectangle $OMP'N$ is completed. Find the curve which is such that the triangle formed by the tangents at any three points P, Q, R is equal to the triangle formed by the corresponding points P', Q', R'.

No solution of this problem has been received.

CALCULUS.

316. Proposed by C. N. SCHMALL, New York City.

$$\int_0^\infty \frac{\cos ax}{1+x^2}dx=\tfrac{1}{2}\pi e^{-a}=\int_0^\infty \frac{x\sin ax}{1+x^2}dx.$$

(From Bromwich, *Theory of Infinite Series*, p. 442, ex. 5, and also from Carslaw, *Fourier's Series*, p. 113, ex. 12.) Prove this by any method.

II. Solution by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Burghfield, England.

Lemma. $u=\underset{h\doteq 0}{L}[\sin h+\frac{1}{2}\sin 2h+\frac{1}{3}\sin 3h+\ldots]=\underset{h\doteq 0}{L}\dfrac{\pi-h}{2}$, the sum being taken between 2π and small values of h, $=\frac{1}{2}\pi$.

$\therefore u=\displaystyle\int_0^\infty \frac{\sin x}{x}dx=\tfrac{1}{2}\pi$, and it is also clear that $\displaystyle\int_0^\infty \frac{\sin ax}{x}dx=\tfrac{1}{2}\pi$.

Now let $U=\displaystyle\int_0^\infty \frac{x\sin ax\,dx}{1+x^2}$; $U-\frac{1}{2}\pi=\displaystyle\int_0^\infty \frac{x\sin ax\,dx}{1+x^2}-\int_0^\infty \frac{\sin ax}{x}$

$$=-\int_0^\infty \frac{\sin ax}{x\,(1+x^2)}dx\ldots(1).$$

Differentiating twice with respect to a,

$$\frac{d^2U}{da^2}=\int_0^\infty \frac{x\sin ax}{1+x^2}dx=U.$$

Multiplying by $\dfrac{dU}{d\,a}$, $\dfrac{d^2U}{da^2}\cdot\dfrac{dU}{d\,a}=\dfrac{U.dU}{da}$. $\frac{1}{2}\dfrac{d}{d\,a}\left(\dfrac{dU^2}{d\,a^2}\right)=\frac{1}{2}\dfrac{d\,(U^2)}{da}$, $\left(\dfrac{dU}{d\,a}\right)^2=U^2+\kappa$,

$$\frac{dU}{\sqrt{(U_2+\kappa)}}=da.$$